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AN OBSERVER BASED BLADE-PITCH CONTROLLER FOR WIND USING FINITE SLIDING MODE IN HIGH WIND SPEED

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Abstract

This paper deals with the problem of designing a robust dynamic output feedback controller for the wind machine. This paper for the first time exploits the designing controller problem of the wind turbines in the presence of time varying delay and uncertain parameters. In this paper, a novel algorithm is proposed which designs a proper controller based on the idea of Lyapunov Krasovskii functional and Finsler's Lemma. To validate the result of the proposed algorithm, comparative simulation examples are given which are two different dimension turbines to investigate the performance of the design methodology as compared to those of previous approaches.

Keywords: Time varying delay; wind turbine; Lyapunov Krasovskii functional; Output feedback controller.

1. INTRODUCTION

Investigation and control of wind machines are frequently encountered in the nowadays studies [1]. Wind power is one of the fastest growing electrical industry and owning the rapid development progress among the other renewable power generation elements.

The large wind turbines take more attentions in recent literatures which are kindly massive structures with enormous blade spans [2]. With the enlargement of wind turbine generating capacity, it is vital to develop feasible, reliable and powerful control strategies in the wind energy conversion systems to achieve maximum power performance.

The control objectives are different when the wind speed varies over its nominal values. When it is below the nominal value, the variable speed controller has been used. The main objective of this controller is to extract the maximum energy form the wind power at its operating point. When the wind speed is above the nominal speed, the pitch controller has been used which its objective is to maintain the output power constant. Since the variable speed wind turbine can produce higher energy with lower component mechanical stress, this type of turbines has become field of increasing interest [3,4]. Some interesting methods for designing simple controllers were developed during the past fifty years [8, 9], such as lead-lag compensation, loop-shaping, PID, and Quantitative Feedback Theory. These methods provides acceptable performances, however, they do not provide a general method for tuning the controller design parameters [7].

Consideration of the delay in the model of wind machines is not usual in the previous studies. However, the existence of the time varying delay in this model is generally clear due to the natural properties of its dynamics. Additionally, hydraulic pressure drive unit in large power wind generation system causes time varying delay to the wind generation system [10, 11].Unfortunately, there is not any study that basically and directly investigates the time varying delay in the wind model equations. Indeed, the controllers proposed so far have been designed without considering any time varying delay in the system's model.

This paper proposes an algorithm to design an output feedback controller for the wind turbine machines in the presence of time varying delay and uncertain model parameters. We briefly present a conventional wind turbine model. Then, a proper controller is designed by the proposed algorithm and applied to the turbine model.. The proposed controller is basically presented based on the idea of Lyapunov Krasovskii functional. This idea is frequently encountered in the previous studies [13-17].

This paper is organized as follows: Section 2 proposes the transfer function model from pitch to tower fore-aft deflection including a time varying delay in the hydraulic pressure drive unit of wind generation system. In Section 3, the main idea of this paper is mentioned which is an algorithm to design a proper controller for the uncertain model in the presence of time varying delay. The simulation examples are presented in section 4 which consists of two examples with different dimension turbines. The simulation results reveal the superiority of the proposed controller. Finally, Section 5 concluded the paper.

2. SYSTEM MODEL

At the operating point, the linear model of blade-tower dynamics is considered to be as follows [12]:

$$f(s) = G_p(s)\beta(s) \quad (1)$$

Where f is the tower fore-aft modal deflection and β is the deviation of the pitch angle from its nominal value. Indeed, equation (1) states that there exists a causal linear equation relating the tower deflection to the pitch angle deviation. The transfer model $G_p(s)$is assumed to have the following form:

$$G_p(s) = \frac{a_2s^2+a_1s+a_0}{s^4+b_3s^3+b_2s^2+b_1s+b_0} \quad (2)$$

The transfer function coefficients $\{a_i\}_{i=0}^2$ and $\{b_i\}_{i=0}^3$ represents the time constant of the wind generation model. The pitch-driven model is under the influence of the hydraulic pressure drive which causes time delay to the generation model [12]. It complexes controller design and analysis stability of the model. According to this effect, the model equation (2) will be modified as follows:

$$G_p(s) = \frac{a_2s^2+a_1s+a_0}{s^4+b_3s^3+b_2s^2+b_1s+b_0}e^{-\tau sn} \quad (3)$$

Where τ is the time delay parameter. Usually, the time delay parameter is considered to be constant and fix in time in the previous studies [12]. This assumption has reduced generality of the model because the delay parameter is under influence of some physical and mechanical model which is varying with time. Hence, the delay parameter is assumed to be time varying in a limited interval as:

$$\forall t:\ \tau(t) \in [\tau_l, \tau_u] \quad (4)$$

Where τ_l and τ_u are the values of the lower and upper bounds of the time varying delay, respectively. It is worth to mention that the time derivation of the delay should be finite due to some physical constraints. Hence, the following assumption is considered:

$$\forall t:\ |\dot{\tau}(t)| \le \tau_D \quad (5)$$

Where τ_D is the upper bound of the time derivation of the mentioned delay parameter. According to the model deviations, time, and physical dependency of the model, the coefficients of the transfer function (3) are considered to have fix but unknown values. Thus, the following equations describe the uncertainty bounds of these coefficients: The values of these coefficients depend on the physical specification of the wind machine and environment parameters. Hence, the lower and upper bounds in equations (6) have been determined such that covers the reasonable deviations of the physical and environment parameters.

$$a_i \in [\underline{a}_i, \bar{a}_i]\ \ for\ i = 0,1,2$$

$$b_i \in [\underline{b}_i, \bar{b}_i]\ \ for\ i = 0,1,2,3 \quad (6)$$

Before presenting the main idea of this paper, the following lemmas are needed to present.

Lemma 1 **[15].**Assume $g(\theta): \mathbb{R} \to \mathbb{R}^n$ is a vector function, Q is a symmetric positive definite matrix and c_1 and c_2 are positive numbers$(c_2 > c_1)$. Then, the following inequality is satisfied:

$$(c_2 - c_1)\int_{c_1}^{c_2} \dot{g}(\alpha)^T Q \dot{g}(\alpha) d\alpha \ge$$

$$\left(g(c_2) - g(c_1)\right)^T Q \int_0^h g(\alpha)^T Q g(\alpha) d\alpha \quad (7)$$

Lemma 2 **(Finsler's Lemma) [16].** Let $x \in R^n$, $Q \in R^{n\times n}$ be a symmetric positive definite matrix and $B \in R^{m\times n}$ such that $\mathrm{rank}(B) < n$. Then, the following statements are equivalent:

i) $\forall x:\quad Bx = 0 \to x^T Q x < 0$
ii) $\exists X \in R^{n\times m}:\quad Q + XB + B^T X^T < 0$
iii) $B^{\perp T} Q B^{\perp} < 0$

Where $B^\perp$ is the null matrix of the matrix B which means $BB^\perp = 0$.

3. CONTROLLER DESIGN

In this section, the main idea of this paper is presented which is to design an output feedback controller for the model (3). The designed controller should guarantee the stability the model for all corner values of model coefficients and considering the time varying delay. For this purpose, a set of mathematical tools is used to design the controller which is mentioned in the following. Before presenting the designing approach, model (3) is transformed into the state space model such as follows:

$$\dot{x}(t) = Ax(t) + B\beta\big(t - \tau(t)\big) \quad (8)$$
$$f = Cx(t)$$

Where $x(t) \in R^4$ is the state vector of this model. The model matrices are also obtained as given below:

$$A(b) = \begin{bmatrix} -b_3 & -b_2 & -b_1 & -b_0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$
$$C(a) = [0 \quad a_2 \quad a_1 \quad a_0]$$

Then, the controller is considered to have the following structure:

Using equations (9), one obtains:

$$\dot{x}_c(t) = A_c x_c(t) + B_c f(t)$$
$$\beta(t) = C_c x_c(t) \quad (10)$$

where $x_c \in R^{n_c}$ in which n_c is the controller order and $A_c \in R^{n_c \times n_c}$, $B_c \in R^{n_c}$ and $C_c \in R^{n_c}$ are controller matrices that are considered as the design parameters. Indeed, these matrices should have design such as the closed loop model is stable. Therefore, in the following the equation of the closed loop model is obtained:

$$\dot{z}(t) = \begin{bmatrix} A(b) & 0_{n,n_c} \\ B_c C(a) & A_c \end{bmatrix} z(t) + \begin{bmatrix} 0_{n,n} & BC_c \\ 0_{n_c,n} & 0_{n_c,n_c} \end{bmatrix} z\big(t - \tau(t)\big) \quad (11)$$

Where $z(t) = [x^T(t) x_c^T(t)]^T$ is the state vector of the closed loop model. For convince, the following notations are defined:

$$\mathcal{A}(a,b) = \begin{bmatrix} A(b) & 0_{n,n_c} \\ B_c C(a) & A_c \end{bmatrix} \quad (12)$$

Based on equations (12), the partial derivation of γ with respect to β can be obtained such as follows:

$$\mathcal{A}_d = \begin{bmatrix} 0_{n,n} & BC_c \\ 0_{n_c,n} & 0_{n_c,n_c} \end{bmatrix} \quad (13)$$

Using definitions (12-13), one obtains:

$$\dot{z}(t) = \mathcal{A}(a,b) z(t) + \mathcal{A}_d z\big(t - \tau(t)\big) \quad (14)$$

Please, note that the system matrices of the closed loop model are uncertain due to the model coefficient uncertainty. For convince, the following notations are used to mention the model uncertainties:

$$\pi_a = \left\{ \begin{matrix} [\underline{a}_0, \underline{a}_1, \underline{a}_2], [\underline{a}_0, \underline{a}_1, \bar{a}_2], [\underline{a}_0, \bar{a}_1, \underline{a}_2], \\ \dots, [\bar{a}_0, \bar{a}_1, \bar{a}_2] \end{matrix} \right\} \quad (15)$$

$$\pi_b = \left\{ \begin{matrix} [\underline{b}_0, \underline{b}_1, \underline{b}_2, \underline{b}_3], [\underline{b}_0, \underline{b}_1, \underline{b}_2, \bar{b}_3], \\ \dots, [\bar{b}_0, \bar{b}_1, \bar{b}_2, \bar{b}_3] \end{matrix} \right\} \quad (16)$$

In the sequel of this section, the designing procedure is proposed which consists of two separate stages. To completely understand the steps of this algorithm, the following lemmas and theorem will be needed to mention.

Theorem 1. Assume there exists symmetric positive definite matrices $P \in R^{m\times m}$, $\{Q_i\}_{i=1}^3 \subset R^{n\times n}$ and $\{R_i\}_{i=1}^3 \subset R^{n\times n}$and also matrix $\{Y_{a,b}\}_{a\in\pi_a, b\in\pi_b} \in R^{5n\times n}$ that satisfy the following conditions (17):

where $N_{d_1}$and N_{d_2} are the number of subdomains of $[\underline{V}, \bar{V}]$ and $[\underline{\omega}, \bar{\omega}]$, respectively, $[\underline{V_i}, \bar{V_i}]$ is the i^{th} subdomain of $[\underline{V}, \bar{V}]$, and $[\underline{\omega_j}, \bar{\omega}_j]$ is the j^{th} subdomain of $[\underline{\omega}, \bar{\omega}]$. The corners of each region are defined as:

$$\forall a \in \pi_a, \forall b \in \pi_b:$$
$$\begin{bmatrix} \phi_1 & \frac{e^{-\gamma\tau_l}}{\tau_l}R_1 & (1-\tau_D)\frac{e^{-\gamma\tau_u}}{\tau_u}R_2 & \frac{e^{-\gamma\tau_u}}{\tau_u}R_3 & P \\ * & \phi_2 & 0_{m,m} & 0_{m,m} & 0_{m,m} \\ * & * & \phi_3 & 0_{m,m} & 0_{m,m} \\ * & * & * & \phi_4 & 0_{m,m} \\ * & * & * & * & \phi_5 \end{bmatrix} \quad (17)$$
$$+He\{Y_{a,b}[\mathcal{A}(a,b) 0_{m,m} \mathcal{A}_d 0_{m,m} \ - I_{m,m}]\} \leq 0$$

Where $m = n + n_c$ and matrices $\{\phi_i\}_{i=1}^5$and Gare defined with the equations (18):

$$\phi_1 = \gamma P + Q_1 + Q_2 + Q_3 - \frac{e^{-\gamma\tau_l}}{\tau_l}R_1 - (1-\dot{\tau})\frac{e^{-\gamma\tau_u}}{\tau_u}R_2 - \frac{e^{-\gamma\tau_u}}{\tau_u}R_3$$
$$\phi_2 = -e^{-\gamma\tau_l}Q_1 - \frac{e^{-\gamma\tau_l}}{\tau_l}R_1$$
$$\phi_3 = -e^{-\gamma\tau_u}(1-\dot{\tau})Q_2 - (1-\dot{\tau})\frac{e^{-\gamma\tau_u}}{\tau_u}R_2$$
$$\phi_4 = -e^{-\gamma\tau_u}Q_3 - \frac{e^{-\gamma\tau_u}}{\tau_u}R_3$$
$$\phi_5 = \tau_l R_1 + \tau R_2 + \tau_u R_3 \quad (18)$$

Then, model (14) will be globally exponentially stable.

$$V_1 = z^T(t) P z(t) \quad (19)$$
$$V_{2,1} = \int_{t-\tau_l}^{t} e^{-\gamma(t-\alpha)} z^T(\alpha) Q_1 z(\alpha) d\alpha \quad (20)$$
$$V_{2,2} = \int_{t-\tau}^{t} e^{-\gamma(t-\alpha)} z^T(\alpha) Q_2 z(\alpha) d\alpha \quad (21)$$
$$V_{2,3} = \int_{t-\tau_u}^{t} e^{-\gamma(t-\alpha)} z^T(\alpha) Q_3 z(\alpha) d\alpha \quad (22)$$

$$V_2 = V_{2,1} + V_{2,2} + V_{2,3} \quad (23)$$

$$V_{3,1} = \int_{-\tau_l}^{0} \int_{t+\alpha}^{t} e^{-\gamma(t-\beta)} \dot{z}^T(\beta)\, R_1 \dot{z}(\beta) d\beta d\alpha \quad (24)$$

$$V_{3,2} = \int_{-\tau}^{0} \int_{t+\alpha}^{t} e^{-\gamma(t-\beta)} \dot{z}^T(\beta)\, R_2 \dot{z}(\beta) d\beta d\alpha \quad (25)$$

$$V_{3,3} = \int_{-\tau_u}^{0} \int_{t+\alpha}^{t} e^{-\gamma(t-\beta)} \dot{z}^T(\beta)\, R_3 \dot{z}(\beta) d\beta d\alpha \quad (26)$$

$$V_3 = V_{3,1} + V_{3,2} + V_{3,3} \quad (27)$$

Then, the main Lyapunov Krasovskii functional is considered to be as follows;

$$V = V_1 + V_2 + V_3 \quad (28)$$

In the following the time derivations of these Lyapunov functions are obtained, successively.

$$\dot{V}_1 = -\gamma V_1 + \gamma z^T(t) P z(t) + z^T(t) P \dot{z}(t) + \dot{z}^T(t) P z(t) \quad (29)$$

Obviously, the time derivation of the first Lyapunov Krasovskii functional will be obtained such as follows:

Using the Leibnitz formula [16], one obtains:

$$\dot{V}_{2,1} = -\gamma V_{2,1} + z^T(t) Q_1 z(t) - e^{-\gamma \tau_l} z^T(t-\tau_l) Q_1 z(t-\tau_l) \quad (30)$$

$$\dot{V}_{2,2} = -\gamma V_{2,2} + z^T(t) Q_2 z(t) - e^{-\gamma \tau}(1-\dot{\tau}) z^T(t-\tau) Q_2 z(t-\tau) \quad (31)$$

For convince, consider the following notation:

$$\dot{V}_{2,3} = -\gamma V_{2,3} + z^T(t) Q_3 z(t) - e^{-\gamma \tau_u} z^T(t-\tau_u) Q_3 z(t-\tau_u) \quad (32)$$

Using equations (30-32), the following equation is obtained:

$$\dot{V}_2 \le -\gamma V_2 + z^T(t)(Q_1 + Q_2 + Q_3) z(t) - e^{-\gamma \tau_l} z^T(t-\tau_l) Q_1 z(t-\tau_l) - e^{-\gamma \tau_u}(1-\dot{\tau})\, z^T(t-\tau) Q_2 z(t-\tau) - e^{-\gamma \tau_u} z^T(t-\tau_u) Q_3 z(t-\tau_u) \quad (33)$$

Also, using Leibnitz formula, one obtains:

$$V_{3,1} = -\gamma V_{3,1} + \tau_l \dot{z}^T(t) R_1 \dot{z}(t) - \int_{t-\tau_l}^{t} e^{-\gamma(t-\alpha)} \dot{z}^T(\alpha)\, R_1 \dot{z}(\alpha) d\alpha \quad (34)$$

$$V_{3,2} = -\gamma V_{3,2} + \tau \dot{z}^T(t) R_2 \dot{z}(t) - (1-\tau_D) \int_{t-\tau}^{t} e^{-\gamma(t-\alpha)} \dot{z}^T(\alpha)\, R_2 \dot{z}(\alpha) d\alpha \quad (35)$$

$$V_{3,3} = -\gamma V_{3,3} + \tau_u \dot{z}^T(t) R_3 \dot{z}(t) - \int_{t-\tau_u}^{t} e^{-\gamma(t-\alpha)} \dot{z}^T(\alpha)\, R_3 \dot{z}(\alpha) d\alpha \quad (36)$$

According to the Lemma 1, the following equations will be obtained:

$$V_{3,1} \le -\gamma V_{3,1} + \tau_l \dot{z}^T(t) R_1 \dot{z}(t) - \frac{e^{-\gamma \tau_l}}{\tau_l}\big(z(t) - z(t-\tau_l)\big)^T R_1 \big(z(t) - z(t-\tau_l)\big) \quad (37)$$

$$V_{3,2} \le -\gamma V_{3,2} + \tau \dot{z}^T(t) R_2 \dot{z}(t) - (1-\tau_D) \frac{e^{-\gamma \tau_u}}{\tau_u}\big(z(t) - z(t-\tau)\big)^T R_2 \big(z(t) - z(t-\tau)\big) \quad (38)$$

$$V_{3,3} \le -\gamma V_{3,3} + \tau_u \dot{z}^T(t) R_3 \dot{z}(t) - \frac{e^{-\gamma \tau_u}}{\tau_u}\big(z(t) - z(t-\tau_u)\big)^T R_3 \big(z(t) - z(t-\tau_u)\big) \quad (39)$$

Using equation (37-39), the following equation is obtained:

$$\dot{V}_3 \le -\gamma V_3 + \dot{z}^T(t)(\tau_l R_1 + \tau R_2 + \tau_u R_3)\dot{z}(t) - \frac{e^{-\gamma \tau_l}}{\tau_l}\big(z(t) - z(t-\tau_l)\big)^T R_1 \big(z(t) - z(t-\tau_l)\big) - (1-\tau_D)\frac{e^{-\gamma \tau_u}}{\tau_u}\big(z(t) - z(t-\tau)\big)^T R_2 \big(z(t) - z(t-\tau)\big) - \frac{e^{-\gamma \tau_u}}{\tau_u}\big(z(t) - z(t-\tau_u)\big)^T R_3 \big(z(t) - z(t-\tau_u)\big) \quad (40)$$

Using equations (29), (33) and (40), the time derivation of the main Lyapunov function can be written as follows:

$$\dot{V} \le -\gamma V + \zeta^T \begin{bmatrix} \phi_1 & \frac{e^{-\gamma \tau_l}}{\tau_l} R_1 & (1-\tau_D)\frac{e^{-\gamma \tau_u}}{\tau_u} R_2 & \frac{e^{-\gamma \tau_u}}{\tau_u} R_3 & P \\ * & \phi_2 & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ * & * & \phi_3 & 0_{n\times n} & 0_{n\times n} \\ * & * & * & \phi_4 & 0_{n\times n} \\ * & * & * & * & \phi_5 \end{bmatrix} \zeta \quad (41)$$

Where $\zeta(t) = [z^T(t) z^T(t-\tau_l) z^T(t-\tau) z^T(t-\tau_u) \dot{z}^T(t)]^T$.

Using condition (41) and based on the Finsler's Lemma, one has:

$$\dot{V} \le -\gamma V \quad (42)$$

The above equation establishes the globally exponentially stability of the model (14).

Remark 1. Theorem 1 proposes a set of conditions to establish the stability of the model (14) with the mentioned model assumptions. The proposed conditions exploits the exponentially stability of the model (14) considering conditions (4-6).

Remark 2. Theproposed conditions of Theorem 1 are not LMI due to the existence of coupling terms between Lyapunov and design parameters in the term $Y[\mathcal{A}(a,b) 0_{m,m} \mathcal{A}_d 0_{m,m} - I_{m,m}]$. It is worth to mention that the closed loop matrices $\mathcal{A}(a,b)$ and $\mathcal{A}_d$ depend on the design parameters A_c, B_c and C_c. Hence, the two stage algorithm is used to solve the conditions of Theorem 1.

As mentioned before, the conditions of Theorem 1 is not LMI and to solve the conditions with LMI toolboxes the following optimization problem is proposed: <u>P1:</u> $min_{h,P,\{Q_i\}_{i=1}^{3},\{R_i\}_{i=1}^{3},YA_c,B_c,C_c}\ h$

$$P > 0$$

$$R_i > 0, Q_i > 0 \ for\ i = 1,2,3$$

$\forall a \in \pi_a, \forall b$

$$\in \pi_b: \begin{bmatrix} \phi_1 & \frac{e^{-\gamma\tau_l}}{\tau_l}R_1 & (1-\tau_D)\frac{e^{-\gamma\tau_u}}{\tau_u}R_2 & \frac{e^{-\gamma\tau_u}}{\tau_u}R_3 & P \\ * & \phi_2 & 0_{m,m} & 0_{m,m} & 0_{m,m} \\ * & * & \phi_3 & 0_{m,m} & 0_{m,m} \\ * & * & * & \phi_4 & 0_{m,m} \\ * & * & * & * & \phi_5 \end{bmatrix}$$

$$+ He\{Y_{a,b}[\mathcal{A}(a,b,A_c,B_c)0_{m,m}\mathcal{A}_d(C_c)0_{m,m} \;-I_{m,m}]\} \leq hI_{5n\times 5n}$$

$$\phi_1 = \gamma P + Q_1 + Q_2 + Q_3 - \frac{e^{-\gamma\tau_l}}{\tau_l}R_1 - (1-\tau_D)\frac{e^{-\gamma\tau_u}}{\tau_u}R_2 - \frac{e^{-\gamma\tau_u}}{\tau_u}R_3$$

$$\phi_2 = -e^{-\gamma\tau_l}Q_1 - \frac{e^{-\gamma\tau_l}}{\tau_l}R_1$$

$$\phi_3 = -e^{-\gamma\tau_u}(1-\tau_D)Q_2 - (1-\tau_D)\frac{e^{-\gamma\tau_u}}{\tau_u}R_2$$

$$\phi_4 = -e^{-\gamma\tau_u}Q_3 - \frac{e^{-\gamma\tau_u}}{\tau_u}R_3$$

$$\phi_5 = \tau_l R_1 + \tau_u R_2 + \tau_u R_3$$

Finally, using the mention preliminaries and Theorem 1, the steps of the proposed algorithm are briefly presented such as follows:

Algorithm:

1) *Set* $k = 0$.
2) *Consider matrices* $A_c^{(0)}$, $B_c^{(0)}$ *and* $C_c^{(0)}$ *to have the random values.*
3) *Solve the optimization problem P1by considering* $A_c = A_c^{(k)}$, $B_c = B_c^{(k)}$ *and* $C_c = C_c^{(k)}$ *to obtain* $P^{(k)}$, $\{Q_i^{(k)}\}_{i=1}^3$,$\{R_i^{(k)}\}_{i=1}^3$*and* $\{Y_{a,b}^{(k)}\}_{a\in\pi_a,b\in\pi_b}$ *and* $h^{(k)}$.
4) *f* $h^{(k)} \leq 0$ *returns* $A_c^{(k)}$, $B_c^{(k)}$ *and* $C_c^{(k)}$ *as the*
5) *solutions of the algorithm.*
6) *Else, solve the optimization problem P1 by considering* $P = P^{(k)}$, $\{Q_i\}_{i=1}^3 = \{Q_i^{(k)}\}_{i=1}^3$, $\{R_i\}_{i=1}^3 = \{R_i^{(k)}\}_{i=1}^3$*and* $\{Y_{a,b}\}_{a\in\pi_a,b\in\pi_b} = \{Y_{a,b}^{(k)}\}_{a\in\pi_a,b\in\pi_b}$ *to obtain* $A_c^{(k+1)}$, $B_c^{(k+1)}$ *and* $C_c^{(k+1)}$ *and* $h^{(k+1)}$.
7) *If* $h^{(k+1)} \leq 0$ *returns* $A_c^{(k+1)}$, $B_c^{(k+1)}$ *and* $C_c^{(k+1)}$ *as the solutions of the algorithm.*
8) *Else if* $|h^{(k+1)} - h^{(k)}| \leq \epsilon$ *finalize the algorithm and returns null.*
9) *Else set* $k = k + 1$ *and go to step 3.*

Remark 3. The parameter ϵ is a threshold value considered to guarantee the convergence and to stop the algorithm if there is not any feasible solution. Therefore, the proposed algorithm will be used to design a proper output feedback controller for the model (14) subject to conditions (4-6).

4. SIMULATION

Two examples are considered in this section to convey the efficiency and performance of the design algorithm in compared to the other previous methods. It is worth to mention that the considered model assumptions of model (14) are not directly used in the previous methods. Hence, to compare the results, the other methods are designed by removing some assumptions. However, the obtained controllers are applied to the original model. The simulation results of this section reveal the superiority of the proposed algorithm in compare to the previous methods.

Example 1. This model is given from paper [12]. The following table presents some physical and environment parameters of this model:

TABLE 1. CONFIGURATION PARAMETERS OF THE FIRST WIND TURBINE

Parameter	value
rotor diameter	$70m$
tower height	$90m$
rated power	$1.5MW$
wind speed	$15m/s$
pitch angle	$0°$

Using the parameter values of Table 1, the transfer function of the wind machine is obtained such as follows [12].

$$G_p(s) = \frac{2.426s^2 - 4.6345s - 147.3}{s^4 + 4.857s^3 + 126.2s^2 + 266.4s + 3659}e^{-.25s} \quad (43)$$

It has been supposed that the coefficients of the above model can have up to 10% error from the nominal values in equation (43). Also, the delay dependent parameters are assumed to be $\tau_l = .2$, $\tau_u = .3$ and $\tau_D = .1$.

Using the proposed algorithm of this paper, the proper controller is obtained which is presented in the following:

$$A_c = \begin{bmatrix} 24.0363 & -121.8684 \\ 883.4176 & -444.4847 \end{bmatrix}, B_c = \begin{bmatrix} -1.2180 \\ -4.4264 \end{bmatrix}$$

$$C_c = [-.7260 \quad -.3313] \quad (44)$$

The following figures show the results of this simulation example:

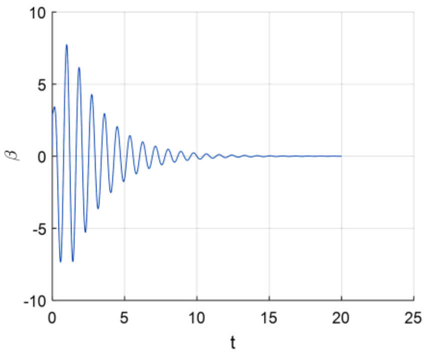

Fig. 1. THE PITCH ANGLE OF THE FIRST SIMULATION EXAMPLE

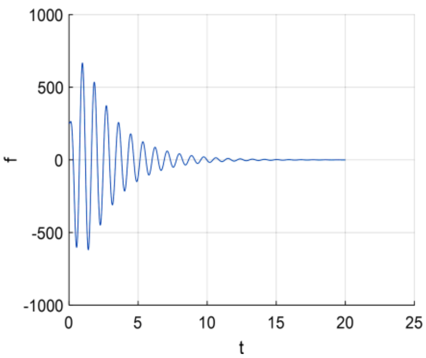

Fig 2. MODAL DEFLECTION OF THE FIRST SIMULATION EXAMPLE

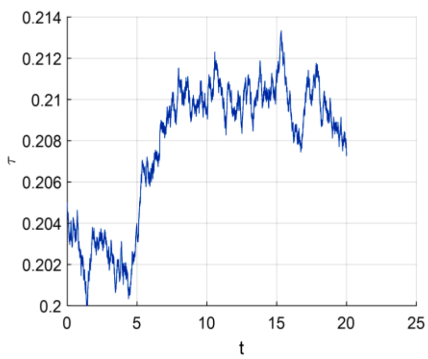

Fig 3. TIME VARYING DELAY OF THE FIRST SIMULATION EXAMPLE

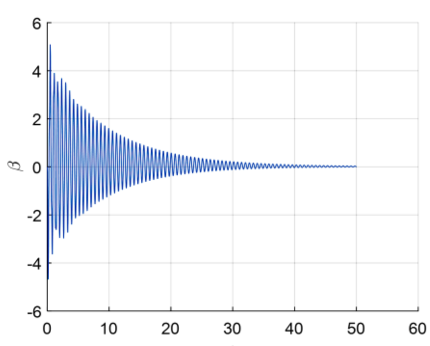

Fig 4. THE PITCH ANGLE OF THE SECOND SIMULATION EXAMPLE

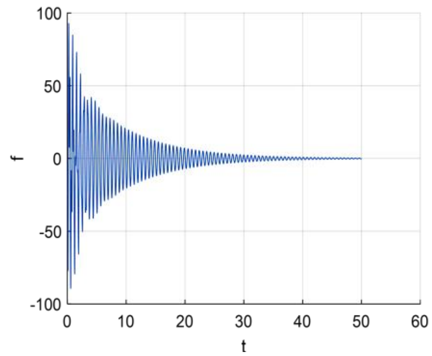

Fig 5. MODAL DEFLECTION OF THE SECOND SIMULATION EXAMPLE

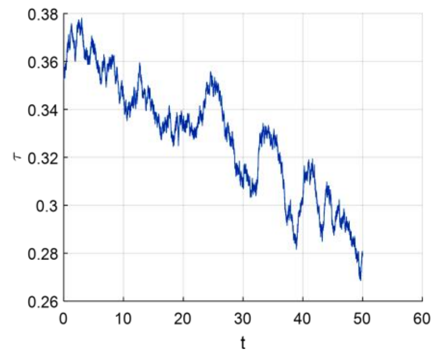

Fig 6. TIME VARYING DELAY OF THE SECOND SIMULATION EXAMPLE

As it can be seen, the designed controller stabilizes the uncertain time varying delay model (43).

Example 2. Consider a wind machine with the following parameters [12],

TABLE 2. PARAMETERS CONFIGURATION PARAMETERS OF THE SECOND WIND TURBINE OF THE FIRST WIND TURBINE

Parameter	value
rotor diameter	$15m$
tower height	$25m$
rated power	$50kW$
wind speed	$15m/s$
pitch angle	$0.75°$

Then, the wind machine has the following transfer function equation which is given from [12]:

$$G_p(s) = \frac{-0.2545s^2 - 0.0647s + 0.9384}{s^4 + 2.28s^3 + 878.5s^2 + 437.7s + 7.7\times10^4} e^{-.25s} \quad (45)$$

Assume the numerator coefficients have 10% error with respect to the nominal values. Also, assume$\tau_l = .1$, $\tau_u = .4$ and $\tau_D = .5$.

Using the proposed algorithm of this paper, the proper controller is obtained which is presented in the following:

$$A_c = \begin{bmatrix} -0.0919 & 0.1601 \\ 0.1224 & -0.8797 \end{bmatrix}, B_c = \begin{bmatrix} 0.4916 \\ 0.3892 \end{bmatrix}$$

$$C_c = [0.9089 \;\; 0.5857] \quad (46)$$

The following figures show the results of this simulation example: This example is also shows the performance of the designed controller to stabilize the closed loop model.

5. CONCLUSION

The problem of design controller for the wind turbine model in the presence of time varying delay and uncertain parameters had been investigated in this paper. The proposed controller is based on the idea of Lyapunov Krasovskii functional and guarantees the globally exponentially stability of the wind turbine model. The simulation results of this paper reveal the efficiency and performance of the proposed controller.

6. REFERENCES

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